"Becoming" a researcher in mathematics education

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As a contribution to the legacy of the Annual Clements/Foyster Lecture, this paper will focus on the theme of becoming a researcher in mathematics education – a fundamental endeavour for MERGA from its foundation. I use the term *becoming* in the socio-cultural sense, that is, how a person develops in their role as an active member of a community. This participation led to the development of an identity – in our circumstance, as mathematics education researchers. Thus, the presentation will not be a research lecture in the traditional sense but, rather, a personal reflection that maps the lived experience of defining my own research program against important junctures of development and a growing sense of *becoming* within the MERGA, and other, communities.

When invited to present the Clements/Foyster Lecture, my first thought was to talk about my research program in the teaching and learning of applications of mathematics – numeracy, mathematical modelling, STEM and the role of digital tools in these areas. This was perhaps a go-to-first gut reaction, and really, I have already spoken about my research in these areas during conferences reaching back to MERGA-16 in 1995. My second thought was that the invitation was an incredible honour, as this lecture was initiated "to honour the foresight of Ken Clements and John Foyster in founding MERGA" (Galbraith, 2014, p. 38). So, the question became, "how could I contribute to this legacy?" Or, to quote David Byrne of the Talking Heads:

> And you may say to yourself, "My God! What have I done?"
>
> …
>
> And you may ask yourself, "well…how did I get here?"

The answers to these questions are far from simple. My background is not one that predisposes an individual to an academic career and certainly not one that would lead me to the position of being one of the few research-only academics in mathematics education. As my beloved mother often remarks, "How does a boy from the working class become a professor?" My good friend, Tom Lowrie, has described me as an outlier in terms of career pathway and academic success (even if moderate success). I am acutely aware, also, that I have not done this on my own. There have been many hands holding me up and giants' shoulders on which I have stood. Given this background and the outcome, it occurred to me that in seeking to understand how indeed "I got here", I might be able to provide insights that point ways forward for others.

So, to answer these questions, I had to reflect on my own development as a researcher in mathematics education, my own *becoming* in the Jean Lave sense of the word, in that "mainly, people are *becoming* kinds of persons" (Lave, 1996, p. 157). This *becoming* spans transformations from student to teacher to researcher, in my case, via a circuitous route. In developing an analytical narrative of this transformation, I will draw on the approach of others who have engaged in the self-study of their own development as mathematics educators/researchers, by reflecting on my personal history through the lens of a theoretical framework (e.g., Krainer, 2008; Tzur, 2001). Given that my formation has been influenced and supported by different communities of teachers and scholars as I stepped into and out of

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different practices, I will adopt a socio-cultural perspective in describing and analysing the development of person-in-practice-in-person (Lerman, 2000).

Conceptual Framework

Studies on the origins of consciousness and knowledge acquisition have tended to focus on individual cognition and intellectual development. The 1980s, however, saw the emergence of theoretical frameworks that placed greater emphasis on the social origins of meaning, thinking and reasoning, a movement Steve Lerman referred to as a "turn to the social" (Lerman, 2000). The origin of such social theories is generally attributed to Vygotsky's (1978) work on child intellectual development. Central to Vygotsky's perspective on the process of intellectual development is the interaction between the learner and a more experienced other working within zones of proximal development (ZPD). The ZPD can be conceptualised as a set of possibilities for development that become actualised when learners interact with more knowledgeable people, for example teachers, and their learning environment. From this perspective, there can be no strict separation of an individual from his or her social environment (Luria et al., 1979), with cognitive development as an outcome of the process of acquiring culture. Thus, the individual and the social must be regarded as complementary elements of a single interacting system (Leont'ev, 1981).

An iconic work in socially oriented theories of learning that emerged at this time was Jean Lave's *Cognition in Practice* (1988). In this book, and later work (e.g., Lave, 1996), she challenged cognitivism and transfer theory in mathematics learning by identifying mathematical *practices* that were appropriated within professions, trades and everyday activities – ways of working and modes of thinking that were far more than the mere application of mathematics acquired from formal education. From her perspective, strategies and decision making associated with the use of mathematics were *situated* in, and products of, the social milieu in which they were employed.

Building on this work, Lave and Wenger (1991) describe learning as a form of apprenticeship where novices are initiated into a learning community, or *community of practice*, through a process termed legitimate peripheral participation. Experts or more knowledgeable peers are responsible for the induction of individuals into the culture of a community, including beliefs, values, modes of discourse, and means and methods of knowledge creation. Judgments about learning are therefore based on the increased range of participation of the learner within the community. Through this participation, an individual moves from a novice towards mastery as part of who they are *becoming* within a community of practice.

> A community of practice is an intrinsic condition for the existence of knowledge, not least because it provides the interpretive support necessary for making sense of its heritage. Thus, participation in the cultural practice in which any knowledge exists is an epistemological principle of learning. The social structure of this practice, its power relations, and its conditions for legitimacy define possibilities for learning (i.e., for legitimate peripheral participation). (Lave & Wenger, 1991, p. 98)

From this perspective, knowledge must be understood relationally, between people, activity, and social contexts. *Becoming* is consequently the degree to which a participant adopts the values, modes of reasoning and discourse practices of a community of practice. This position is, therefore, a direct challenge to the notion that knowledge construction takes place by the transfer of decontextualised mental objects from one individual to another.

Building on his work with Lave, Wenger (1998) extended the notion of *becoming* to the formation of an individual's identity within a community of practice. As a consequence, an

individual's identity within a community is strongly influenced by their personal affiliation with its beliefs, values, modes reasoning and processes of knowledge production.

> Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming - to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity. (Wenger, 1998, p. 215)

Thus, the identity an individual establishes within a community of practice is dependent on how they act and interact with others – the role they play as part of a community. At the same time, this identity is influenced by the community itself and the individual's sense of belonging to a community. The relationship between the individual and the community is thus reflexive – one evolving with the other.

Lerman (2000), in offering a critique of these ideas as they relate to mathematics education, raises individuality and agency as issues not fully accommodated in the thinking of proponents of situated and social understandings of learning. How, for example, are they able to realise their own goals within an existing community of practice? In response to this dilemma, he makes the observation that a person's goals are already aligned with a community when they step into a practice because this is the reason they choose to become part of a community of practice. This means that not only is the person becoming in the practice but that the alignment of goals means that the practice is becoming in the person. Consequently, he suggests that the unit of analysis for socially orientated studies in mathematics education should be extended to person-in-practice-in-person.

Since this time, the "social-turn" in mathematics education has been extended to incorporate the role of cultural practices, institutional contexts, personal histories, beliefs and values in attempting to understand and describe interactions central to teaching and learning (Goos, 2014). Valsiner (1997), for instance, reconceptualised Vygotsky's zone of proximal development (ZPD), to include two additional *zones* that accommodate both the influence of social settings and the goals and actions of individuals – the zones of free movement (ZFM) and promoted action (ZPA). Within this new construct, the ZPD is a space that defines an individual's potential development, the ZFM was conceived as the ways in which an individual is permitted to act within a context, and the ZPA identifies the conditions within a situation that promote action.

While Valsiner's zone theory was conceived as a theory of child intellectual development, others have extended its use as a tool for understanding human development in other areas. Goos (2013), for example, has interpreted the notion of development more broadly:

> …I take ''development'' to mean more than the formation of higher mental functions in children; instead, it refers to the emergence of new domains of action and thinking and new cultural frameworks that organise a person's social and psychological functioning. (p. 523)

A broader perspective on Valsiner's zone theory has underpinned research within education including students as learners (e.g., Blanton et al., 2005), teachers as learners (e.g., Goos, 2014; Goos & Bennison, 2019; Geiger et al., 2017) and teachers' numeracy identities (e.g., Bennison, 2015). Goos and Bennison (2019) applied the principles of Valsiner's zone theory to the development of teacher educators, attempting to understand how they learn within contexts defined by opportunity and conditions. Through this work, insight was developed into how teacher educator identities develop, and how teacher educators' opportunities to learn can be improved. From this perspective, the ZPD represents the possibilities for development of teacher educators' knowledge and beliefs. This includes the

knowledge of mathematics and pedagogy for teaching, how new teaching practices are learned, and their beliefs about which teaching and learning practices are effective. Within the context of teacher educators' professional environment, the ZFM can be constructed as both external constraints and an individual's own interpretation of related limitations or affordances. Such affordance and constraints include curriculum and assessment requirements stipulated by professional accreditation authorities, access to teaching resources, and the beliefs and expectations of prospective teachers. A teacher educators' ZPD relates to how an individual's goals and actions can be promoted or inhibited by features of their environment or the actions of others, such as their peers or institutional leaders. In the case of teacher educators, a ZPA might include academic structures, recognised markers of career development and promotion, and access to accomplished mentors.

The ZPD, ZFM and ZPA form a complex that represents the dynamic interaction between possibilities and limitations. Teacher educators' learning is thus the interaction of their development potential and their interpretation of opportunities for, and constraints on, progressing professional goals.

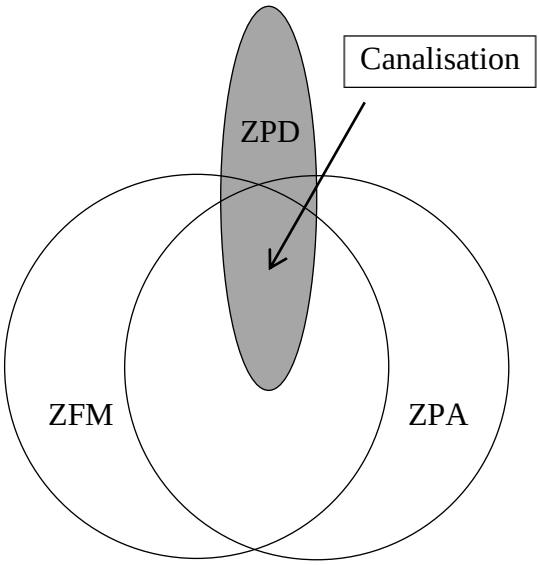

Figure 1. Canalisation of the ZPD

The influence of the intersection of the ZFM/ZPA (Figure 1), on what it is possible to promote within what is permitted, is known as the *canalisation of development* within the ZPD (Blanton et al., 2005; Oerter, 1992). Thus, canalization is how development is shaped under the dynamic influence of the ZFM & ZPA. This means that even though there are constraints individuals retain agency and are not just passive participants.

While Goos has referred to the development as teacher education researcher in her discussion of how teacher educators learn (e.g., Goos, 2008), and others have described the skills and attributes required by educational researchers (e.g., Boaler et al., 2003), there has been limited discussion specific to the development of researchers in mathematics educations across the span of a career. In the remaining part of this paper, I will attempt to provide some insight into this theory/practice gap by drawing on noteworthy junctures of my own development (or failure) as a researcher to speculate on how these contributed to

my understanding of personal and collective history, enculturation, and identity development as a researcher.

Origins

History precedes us. I was one of five children born into a working-class family in Brisbane. My father went to work as a flower boy immediately after finishing primary school, eventually finding an apprenticeship as a wood machinist, which he stuck with until retirement. My mother left school before the end of Year 6 and worked in a string of jobs until finding employment as a seamstress – something she still practices today for friends and family.

For whatever reason, I was good at school mathematics and science. Although they weren't quite sure of the implications, my parents encouraged this interest. I was the only child in the street who owned a microscope, a Christmas present during my primary school years. I'm sure I took on the role of suburb odd bod, sitting on the footpath studying whatever insects I could find to study. My parents' support came out of an understanding that the opportunities in life were afforded by education – my father was determined that all his children would complete Year 10! And so we did. I won a Commonwealth Scholarship which allowed me to go forward to Year 12, only one of five in my cohort. I was already exceeding my fathers' expectations! I remained good at mathematics, although I wasn't always the best student – there were too many other things to do, cricket, rugby and school parties! Towards the end of Year 12, my father had set up a job for me in a bank. This sounded fine to me, but a teacher contacted my parents to say I should consider going onto university. They weren't sure. No one in my extensive extended family (my maternal grandmother had 13 children) had ever done so and it was about time I started earning my keep. The teacher explained that there was a tertiary assistance scheme for those whose parents' combined income was below a particular threshold. We were well below. Always supportive, my parents sat me down and asked if I'd like to go to university. "I suppose so" I said. And so, I found myself enrolled in a Bachelor of Science in the second intake of the newly minted Griffith University.

I majored in physics and physical chemistry, with widely varying levels of achievement across the degree. I didn't quite get the game of tertiary study at that stage. But university provided me with the opportunity to take courses in the philosophy of science, something quite exotic for a boy from the working class. I found it fascinating. Seminars revolved around types of thinking I had never encountered before – especially discussions about how knowledge was generated. In *Conjectures and refutations: The growth of scientific* knowledge, Karl Poppers (1963) argued that knowledge is not simply discovered but developed through a process of conjecture and refutation. This set me back on my heels! This was followed by Thomas Kuhn's (1962) *Structure of Scientific Revolutions*, in which he outlined the paradigmatic nature of knowledge creation in science; an epiphany that set me on the path of fallibilism for life.

I had initially entered the course thinking I would complete a dual qualification – a Bachelor of Science and a Diploma of Teaching – but I had found the demands of tertiary study demanding and did not believe I had the discipline to continue for another year beyond the BSc. I completed the qualification and went looking for a job. I spent the next 18 months working on the line gangs for Telecom until I was successful with an application for a research assistant within the Department of Engineering at the University of Queensland – a dream job!

While it might be premature to think about the affordances and opportunities at this stage of my life, especially in terms of ZPD, ZPA, and ZFM, there are traces of this prism I can see from the distance of time. I had an interest in mathematics and scientific study from an early age (ZPD), which was supported by my parents, despite their limited educational opportunities (ZFM). They also provided me with resources to encourage my mathematical and scientific interests (ZPA). My opportunities in education far exceeded those of my parents as I was able to continue in school through to Year 12 and then to university (ZFM/ZPA complex). From this point, however, my belief that I could not study beyond a BSc was a self-imposed ZFM. My way forward was to create a ZPA that saw me on a trajectory from my current ZFM, working as a linesman for Telecom, to a role in a university that supported the research of others.

My identity, through this period of my life, was subject to constant change – but who was I *becoming*? I had been successful at school, but the cohort to which I belonged went their separate ways after graduation. My participation in academic life at university was not to a depth where I felt engaged enough in the community that I wished to continue. Yet, it would seem that my goals aligned with research work in universities sufficiently to step into a new community within an engineering department. Would this take hold?

Changing Course – a Teaching Career

I enjoyed the work in the Department of Engineering, most of which was focused on the building and testing of a wind tunnel. I was able to use a little of my mathematical and scientific capabilities but after 18 months and many hours sitting in front of a small, heated metal filament used to measure the characteristics of wind flow in the tunnel as part of the process of calibration, I began to think that, perhaps, there were more exciting ways to make a living. I had a number of friends who had completed their dual qualifications at Griffith University and were now teachers. They were enjoying the challenges of the profession, so I decided to join them – and enrolled in a Diploma of Education at the University of Queensland. During the course I met two people who would be very strong influences on my life, Marjorie Carss and Peter Galbraith, both of whom were teaching in the program.

I was introduced to the idea of pedagogy! I had entered the course believing that teaching was only a matter of telling or showing others how to do something. There was apparently much more to it! I was intrigued that there might be different approaches to teaching that should be implemented depending on the context – there was no single right way, an echo of fallibilism. During practicum I became aware of the complexity of the classroom and learnt that my best lessons were those that were approached as a problem to solve. I also came to understand that documenting what worked and what didn't (as stipulated by Marjorie and Peter), and reflecting on why, made a difference to the success of follow-up lessons. Then, before I thought I was ready (but whoever is), I had finished the course and been invited onto the staff at the school where I had completed my final practicum. It seems that the principal thought I had some potential.

In those first years, I experienced all the ups and downs that most early career teachers encounter. But I slowly established myself within the school as I worked on improving my teaching in mathematics, junior science, and physics. I was never left to my own devices as I could always depend on teaching colleagues for advice and there was also ongoing contact from Marjorie and Peter. It was Marjorie who convinced me to enrol in a Bachelor of Educational Studies (BEdSt). I didn't really know why this was a good thing to do, but Marjorie was so sure! After completing the BEdSt, there was a pincer movement from both

Marjorie and Peter that resulted in my enrolment in a Master of Educational Studies (MEdSt). It helped that all of these courses were free at the time.

After starting this course, however, I decided to spend some time in Europe – like many young Australians. I resigned from my position and headed to England. To make ends meet, I took up a supply teaching position at an inner-city London school at the turbulent time of school amalgamations. Schools at this place and time could be described as cheerful but violent. Students, in the main, came from low-income backgrounds, many with dysfunctional families. Few of the parents I met had aspirations for their children's education beyond finishing O-levels. Things were tough, students were difficult to manage, and staff were on occasion assaulted. Almost in contrast, the school was well-resourced with the quality and range of available teaching materials better than those I had access to in my first teaching post. These resources were aimed at developing mathematical competence alone, with little attention to how this might be applied to problems in students' own lives. Without engagement, however, little learning was possible. This experience helped me understand the outcome of disadvantage. It also convinced me of the need to teach mathematics and science in a way that connected with students' lived experience.

The European adventure concluded, and I returned to teaching in Australia. My MEdSt awaited me. I had formulated the idea for my thesis.

It would appear my ZPA was oriented towards a life related to learning and a connection to research was apparent through my employment as a research assistant in a Department of Engineering. Despite working "out-of-field" for a period of time, I was drawn to education, initially as a way of making a more interesting living, but I was open to changing the direction of my life. My ZFM was fashioned by people who became mentors. They provided advice and support that led to my development as a reflective practitioner and further study in education. I was fortunate to have the opportunity for further study without the deterrent of paying fees. My engagement with a preservice program in education, ongoing encouragement from mentors, and experience in the classroom in two different countries provided the impetus for ongoing professional learning – related to both my teaching practice and further formal education (ZPA).

I was being drawn into a community that I did not yet fully understand, but my goals seemed to be aligning. My identity had changed from that of research assistant to teacher. At the same time, a new identity was developing, that of educational researcher, evident in my enrolment in a Research Masters program. However, the identities of teacher and researcher were separate – teaching was my career and focus, while research was something I did out of interest. I was now participating, in a peripheral sense, in two communities of practice. Although there were overlaps, there were different modes of meaning making, reasoning, and knowledge generation to appropriate and reconcile. I can clearly remember being surprised at the differences between discourses as I negotiated my role in these different communities. What was my role as person-in-practice-in-person?

Teacher and Researcher

After returning to Australia, I was successful in an application for a teaching position in a significant city just outside of Brisbane – Ipswich. The school was not unlike that in which I had been worked in London, with many families suffering some form of disadvantage. A significant number of families were from various trouble spots throughout the world, with the students' parents moving to Australia to give their children a better life. The experience only confirmed my conviction that making mathematics relevant to students was key to their engagement and success. This time coincided with work on my MEdSt thesis, *A study of the*

mathematical problem-solving behaviours Year 11 students solving application problems, with my principal supervisor, Peter Galbraith. In this work I was searching for a way to provide students with the type of feedback they needed to improve the way they addressed problems in the real world through mathematics – consistent with my belief that students needed to find mathematics relevant to their lives. This was also a time of marriage and children; the thesis took an age to finish. And I thought I was done with further study!

It was around this time that Marjorie convinced me to attend my first MERGA conference, held in Brisbane in 1993. While I found some aspects of the conference interesting, it appeared to be a combative environment where egos were put on display with abandon. People argued about what I saw as minor points and few took the time to include me in discussions. I decided I would not attend again. An upside of the experience, however, was a presentation by a young researcher named Merrilyn Goos. I thought she made some sense – and she won an Early Career award as an outcome of the presentation!

Three years after returning to Australia, I was successful in securing a Head of Mathematics position at a new school. Marjorie Carss was also encouraging me to make a contribution to the work of mathematics teacher professional associations – first, editor of the Queensland Association of Mathematics Teachers (QAMT) journal and eventually president. As president of QAMT, I found myself as chair of the steering committee for a major national initiative – the National Professional Development Program (NPDP) aimed at improving teaching and learning in Australian schools - a daunting experience for someone with no experience in leading state-wide initiatives. I was also contributing to state-wide committees related to curriculum development and assessment. Marjorie continued to provide advice about how I should shape my career – and I found myself as president of the Australian Association of Mathematics Teachers! There was enough to do. Life was busy and I had a clear direction. I thought I had liberated myself from the demands of further study…but then I was dragged back again!

Peter Galbraith rang. He said there was a young researcher he thought I would enjoy talking to. At that time, I had developed a somewhat cynical attitude toward educational researchers. There had been a number of visits to see what we were doing in our school's Mathematics Department – it had gained some notoriety in the state. They had typically harvested data and left, never to be heard of again. This engagement felt like I was putting in significant effort with no return. But because it was Peter, I agreed to take a call. Some days later, the call came…"Hello, my name is Merrilyn Goos. Peter Galbraith said we should talk". So we did. Merrilyn had begun a PhD study in which she was recruiting secondary school teachers for a project in mathematical problem solving and metacognition. Merrilyn talked with such enthusiasm about her research that I was convinced (with some reluctance) to participate in the study. This began a series of nearly weekly visits from Merrilyn over a period of close to three years.

Merrilyn was different to other researchers I had encountered previously – she was genuinely interested in what I had to say, regarding research as a joint venture with teachers and not something that was done to them. After observation sessions, Merrilyn was never critical, she merely wanted to know why I had taken particular approaches to instruction. I had been a reflective practitioner for some time, but this was an extra pair of eyes that helped me go deeper into the reasons that underpinned my classroom decision-making. In these circumstances, having a researcher in the room was not a burden – it was a serious advantage! What I hadn't realised when agreeing to participate, was that the research was part of an ARC award that Peter Galbraith, Merrilyn, and others had secured. This meant there were publications to be generated! Consistent with Merrilyn's approach to

researcher/teacher collaboration, I was invited to join the writing team in instances when data had been collected from my classroom. Some of this early work (e.g., Goos et al., 2000; Goos et al., 2003), related to the affordances and constraints of technology in promoting collaborative problem solving, remains some of my most highly cited.

I have previously described a pincer movement that had saw me return to study and research in education. This time Merrilyn had established a foundation on one flank by drawing me into her research and co-authorship, while Peter made advances from the other. He had been pleased with the quality of my MEdSt and encouraged me to develop a MERGA paper and to nominate for the Practical Implications Award (PIA). Peter provided advice through rounds of drafting and redrafting, and then off it went. It was successful! Merrilyn meanwhile had insisted I co-present with her at the next MERGA conference. It seemed I had no choice by this stage, and so I had to find a way there – MERGA 18 held in Darwin in 1995. In the PIA paper (Geiger, 1995), I presented a framework for providing feedback to students engaging with applications of mathematics to real world problems. The paper I presented with Merrilyn (Goos & Geiger, 1995) reported on a case study of metacognitive activity and collaborative interactions in a mathematics classroom – my classroom. I can't say I was hooked, but I could see no way out.

It would seem my ZPD was expanding, firstly though Marjorie's encouragement to become engaged with state-wide and national initiatives through participation in teacher professional associations (peripheral participation). At the same time, Merrilyn and Peter had opened up possibilities for involvement in educational research. My ZFM was defined by access to established and promising researchers and my school was supportive of my involvement in their project. I was increasingly becoming involved in communities that engaged with national initiatives in teacher professional learning and those that conducted research in the teaching and learning of mathematics (ZPA). Marjorie's, Merrilyn's and Peter's differing influence, as knowledgeable others, was impacting on my ZFM/ZPA complex, guiding me into new ways of *becoming*. There was a flame and I was the moth.

Through this time, my identities as a teacher and researcher were being reinforced through participation in two different communities of practice. However, other identities were emerging through living life, husband and father, and by participation in a new teacher professional association community. Marjorie was shaping my ZPA through her introduction to the teacher professional development community, with Peter and Merrilyn helping to induct me into research – a different ZPA. At this stage, both were within the constraints of my ZFM. However, was I being pulled in too many different directions?

"Now It's Your Turn"

Merrilyn finished her PhD. It was a wonderful piece of work and provided the basis for articles in the best journals in mathematics education, Educational Studies in Mathematics and the Journal for Research in Mathematics Education. She was on her way! Again, I thought there was a moment when I could escape, but then Merrilyn asked me for coffee and said, "now, it is your turn". It took a little while to agree but I had very much enjoyed working with Peter and Merrilyn, and they convinced me I had something to contribute. And so it came to pass, with Peter and Merrilyn as supervisors. Merrilyn continued to come along to my classes – the extra eyes were invaluable, and the study began well. After the two years, however, things slowed down. The weight of all I had taken on, including the arrival of additional offspring, took its toll. No one was ever able to identify the malady, but I had to stop both work and study. Through this time, however, support was never far away. I was encouraged to do what I could when I could. Slowly I could do more, and eventually, I made

my way back to work and to research. I will be forever thankful for the unwavering support I received at that time from friends and colleagues. They know who they are.

The episode lasted for close to 12 months in its severest stage and for close to five years in all. I could have walked away at any stage but the connection to ideas and the community had become strong. The need to be involved in research was now a part of my identity, and so I was drawn back to the practice - person-in-practice-in person – despite the constraints of poor health (ZFM). It wasn't so easy to get away. After the worst, I began to pick up the threads of my PhD and I received support to present tentative findings at my first MERGA conference after a brief hiatus. I began to understand that involvement in research was now a part of who I was and that needed to sit with my love of working with students. These separate identities were about to reconcile. It was around this time that Peter retired (2003) and Merrilyn took on the responsibility of principal supervisor for my PhD.

Merrilyn and I have written about our work together during this phase of our collaboration (Geiger & Goos, 2006; Goos & Geiger, 2006). These publications took the form of a conversation between two different types of researchers where power and authority were shared in recognition of different types of expertise. But by now I had fully committed to completing my PhD, a very hard thing to do while working in a school – my goals had changed as had my way of thinking about mathematics education. Thus, there was a developing mismatch between my identity and that of the role of a mathematics coordinator within a school.

A friend sent me an advertisement for a Lecturer B position at Australian Catholic University – a relatively new institution that emerged during the transformation of institutions of higher education during the Dawkin's reforms of the early 1990s. I applied for the job and was interviewed by Elizabeth Warren and Tom Cooper. To my surprise, I was successful.

The decision to pursue a PhD meant that my ZPD was about to be extended. In time, my opportunity to complete was facilitated by a change in working circumstances (ZFM) and my own determination to do so (ZPA). Research was about to be part of my responsibilities, not just a "hobby", an essential component of my ZFM, although illness limited my progress for a time. Merrilyn's and Peter's support were a key influence on my ZFM/ZPA complex and identity formation, as this was guided towards further involvement in research, as was the formal requirement to conduct research within my new academic position. Co-authorship was a particularly influential factor in promoting my progress as a researcher.

There were further incremental shifts in identity. My more active participation in the educational research community was disrupting my singular engagement with my role as a teacher. I was now seeking alignment with goals that had changed over time and a new community of practice – this had implications for the person-in-practice-in-person – who was I and to which practice did I belong? Merrilyn and Peter were helping me bridge into the mathematics education community. But at this time, it was still a leap of faith.

Choosing Something Else – A Mathematics Teacher Educator/Researcher

Life had changed again. I was now responsible for the preparation of teachers, principally in mathematics but also in curriculum and assessment – nine courses as lecturer-in-charge in a year. The level of regulation was considerably higher than in a school – course outlines, advanced and detailed notice of assessments, and accreditation considerations. There was a lot to learn, and there was that PhD to finish! Life as a teacher educator/researcher was complex.

I did find a way to complete my PhD (Geiger, 2009) and to write. At first, it was mainly conference papers and book chapters – typically collaborations with more experienced researchers – but slowly I began to take the lead. It was not always smooth sailing, however, with as many rejections as successes in my attempts to publish in high quality outlets. I remember being shattered when it took nearly two years for one manuscript to be rejected! It has never been published as there were other events that overtook me.

A period of study-leave in Giessen, Germany, working with Professor Rudolf Straesser in 2010, provided the space I needed to focus on academic writing and begin to think about funding applications. This was a productive period for publication (4 journal articles and a book chapter). The visit to Giessen also established an ongoing research collaboration with Rudolf that continues to this day (e.g., Geiger & Straesser, 2015; Geiger, Delzoppo, et al. 2021).

Upon returning to Australia, I started attracting additional administrative responsibilities, secondary program coordinator and then, deputy Head of School (Research). This heightened the challenge of maintaining a research identity as teaching loads, administration and research all had to be kept in balance. I made sure that there was at least one writing day a week. This did not mean I put less effort into teaching. This was still central, but I had started to think about how these two different aspects of my identity could be better reconciled. I had begun to think more deeply about the nexus between research and teaching and worked with others on a project related to providing technology based support and resources to students while on practicum. This project, WebCT as a pedagogical resource and communicative tool for use in the professional experience program, was recognised nationally via an ALTC Citation in 2009.

About this time, my Head of School asked me to think more about how to take others forward – I think she was suggesting that I should do more than just think about myself! I took the advice to heart and applied for a number of internal grant opportunities, including others in the applications. One related to the potential of computer algebra systems with Merrilyn and Rhonda Faragher (Geiger et al., 2010) and another related to the collaborative use by teachers of video stimulated recall techniques to improve numeracy teaching practice (e.g., Geiger, Muir, et al., 2016). The first was supported by the Mathematics and Literacy Flagship at ACU, which was led by Doug Clark and the second was supported by an Education Faculty grant. I was also asked to lead a *research support team* for members of the school of Education in Queensland which provided funding for the engagement of a senior researcher in a consultancy role – I asked Robyn Jorgensen. Each of these small grants provided an opportunity to gather data, and the mentorship provided by Robyn promoted our publication capabilities.

I also decided to contribute in a more substantial way to the mathematics education community via MERGA, and successfully nominated for the role of Secretary on the executive (2009-2012). I served under two Presidents – Judy Mousley and Merrilyn Goos. My learning during this opportunity was about the scope of activity in which researchers could be involved, publication, conferences, development and, of course, leadership in the field.

Further leadership opportunities were also emerging. After a selection process, Doug invited me to take up the role of Deputy Director of the Mathematics and Literacy Flagship.

Through this period, I continued to work with Merrilyn on a series of projects related to improving numeracy teaching practice within schools based on a model for numeracy for the 21st Century. The model brought together the dimensions of context, mathematical knowledge, dispositions, tools, and an evaluative element, a critical orientation, for the first

time (e.g., Goos et al., 2014). Shelley Dole and Anne Bennison worked with us on these initial projects which led into a successful ARC Discovery application with Helen Forgasz (2012-2015). I learnt much during this time from established researchers in the field about managing large projects – how to approach schools for recruiting purposes; how to work with education systems as well as teachers in schools, effective practices in data collection and achieving as well as analysis. And there were more opportunities to write. Not many academics from ACU had been involved in ARC funded research at that time and I was determined to take every opportunity to support the project, taking the position that if I couldn't contribute as fully to the study as others on the intellectual plane at that stage, I could make up for it with sheer hard work. This program of research has led to significant publications as different perspectives on numeracy practice emerged, including the use of the numeracy model as a scaffold for planning numeracy lessons across the curriculum (Goos et al., 2014); auditing curriculum for numeracy opportunity (Goos et al., 2012), the nature of numeracy (Geiger, Goos, & Forgasz, 2015), the design of numeracy tasks (Geiger et al., 2014), numeracy readiness of pre-service teachers (Forgasz et al., 2015), role of technology in effective numeracy practice (Geiger, Goos, & Dole, 2015; Goos et al., 2013), and development of numeracy identity (Bennison, 2015). The model that underpins all of this work has received international recognition as a holistic approach to enacting numeracy. For example, it has been included as a framework that informed the development of the PIAAC Cycle 2 assessment framework: Numeracy (Tout, et al., 2021). It also received a MERGA Research Award in 2017. This work is ongoing (e.g., Bennison et al., 2020), with further opportunities to explore in this space with good friends and colleagues.

My ZPD had now changed to accommodate the demands of a mathematics teacher educator – teaching, research, and service. The challenges associated with balancing these demands provided constraints within my ZFM. Further constraints included the standard required for publication in high quality international journals. Involvement with the Numeracy Across the Curriculum program, however, as well as mentoring by established colleagues and a period of study leave, had canalised my development in research and strengthened my connection to relevant communities of practice – strong positive influences on my ZPD. The ZFM/ZPA complex, at this time, was enabled by my development as a researcher even while meeting the many demands of a teaching/research academic – two separate but interrelated communities of practice and associated ZFM/ZPA complexes. The deeper enculturation into educational research was transforming my role as novice into fuller participation in a national community of practice in mathematics education, and I was taking my first steps into the international community. I had also taken steps towards the mentoring of others in a research community of practice, shifting my identity from that of complete novice towards mastery. Each of these developments were contributing to further transformations of my identity – becoming more fully immersed in the research community and while maintaining focus on other aspects of who I was professionally. I had now developed the belief and confidence that I could be a successful researcher but doing everything well was becoming harder, there were only so many hours in a day!

A Broadening Role in the Mathematics Education Community

I had another opportunity for study leave at the beginning of 2014. This time I chose to visit Gabriele Kaiser from Hamburg University in Germany, Katja Maass in Freiburg, and Peter Freid and Jonas Arleback in Linkoping, Sweden. All were part of the mathematical modelling community and connected with my research interest in the teaching and learning of mathematical applications. These visits resulted in further publications, in the short term

or over a longer period of time (e.g., Geiger, Ärlebäck, et al., 2016; Maass et al., 2019; Cai et al., 2014).

During this time, I also wrote drafts for ARC DECRA (for early career researchers) and Discovery Awards. The former was successful and the later, while receiving encouraging reviews, was not supported by the ARC. The focus of the DECRA was an extension of work I had been doing with colleagues, this time looking at the processes teachers engaged when designing numeracy tasks for implementation across the curriculum. Through this study I developed a framework that outlines how numeracy task design takes place through the processes of identification or archiving ideas (looking, seeing, noticing), the shaping of a task to fit the classroom circumstances in which it was to be implemented, and the actualisation of a task in a classroom through a well-considered pedagogical architecture (e.g., Geiger, 2016; Goos et al., 2019). This work provoked further thinking about the role of the critical aspects of numeracy and how these could be actualised by teachers in the classroom through the structure of tasks, measured responsiveness, and forms of questioning (e.g., Geiger, 2019).

Because of the DECRA, I was now being noticed within the university, no one at ACU had been successful previously. I was invited to become a member of the newly formed Institute for Learning Science and Teacher Education, an initiative aimed squarely at establishing research at ACU as world class. I was now a research-only academic. While this provided time to think and write, it corresponded with an increasing number of invitations to collaborate with others on national projects – the *Opening Real Science* (2013-2016) project led by Joanne Mulligan and supported by a range of colleagues from very different backgrounds in mathematics, science, and education (e.g., Geiger et al., 2018), the *Building an evidence base for national best practice in mathematics education* (2015-2016) project, sponsored by the Office of the Chief Scientist and led by Rosemary Callingham, in which I worked with many good colleagues in mathematics education nationally (Geiger et al., 2017). There was further success with an international funding application to the Australian Universities-German DAAD Joint Research Cooperation Scheme (2017-2018), which provided opportunity to work with Jodie Miller and Jill Fielding-Wells as Early Career Researchers on a collaborative project with German colleagues from Darmstadt University led by Regina Bruder. And then, a revision of a previously unsuccessful ARC Discovery application with Gloria Stillman, Jill Brown, Peter Galbraith and Mogens Niss (e.g., Geiger, Galbriath, et al., 2021) was awarded funding for 2017-2019. This focus of this project was on identifying enablers of mathematical modelling from both the perspectives of instruction and learning. Each of these projects provided opportunity to extend ideas within research themes I had been working on for some time – quality teaching and learning through a focus on task design and implementation, applications of mathematics, and the role of digital tools in enhancing instruction. But there was a lot to do! I had learned, through these times that the contributions of support staff make a project work. The contributions are sometimes downplayed by researchers – at their peril! I had learned that leading research was about more than grant capture and publications (although these aspects are important) – it is also about leading people – another identity.

More recently, a collaboration with Sharon Fraser (UTas), Kim Beswick (UNSW) and members of the mathematics education community, led to a successful tender for the Principals as STEM Leaders project (2018-2020) sponsored by the Department of Education, Skills and Employment. An important aspect of the project to date has been a framework of capabilities required by principals, teachers, students, the community, and researchers to promote positive STEM learning cultures within schools. The development of

this framework drew on the dimensions of the model for 21st Century numeracy, instigated by Merrilyn and further developed through collaboration with other colleagues over 15 years. Ideas build on themselves over time.

Robyn Jorgensen continued as an informal mentor beyond her role as a consultant on the research support group in my school, inviting me to put my name forward for a role as Associate Editor of the Mathematics Education Research Journal (MERJ). Robyn was the Editor-in-Chief. I was flattered but was I good enough?! It was a steep learning curve between 2013 and 2018, with a period as Acting Editor-in-Chief. There was much more to publication when looking from the other side of the process – managing reviews and reviewers, developing consistent feedback across submissions and working on my own understanding of what is required in a quality publication. This experience and a maturing publication record led to an invitation to act on the Editorial Board of the International Journal of Science and Mathematics Education as well as three Guest Editorships of ZDM - Mathematics Education.

Other opportunities for international collaborations were now opening up. I was awarded the Giovani Prodi Guest Professorship at Wurzburg University, Germany (2018-2019) from an international field of 50 scholars. This experience has been the foundation of an ongoing collaboration with Hans-Stephan Siller and his team. My ongoing role is to collaborate with Stefan's team on the internationalisation of their research, in the first instance through publication (e.g. Siller et al., under review), leading to funding applications.

I am currently working with an international team on the *Cycle 2 of the Programme for the International Assessment of Adult Competencies* (e.g., Tout et al., 2021). This work has drawn on our numeracy research, and that of others, especially the critical aspects of what it means to be an informed and active citizen. I am also working on another international project with Iddo Gal (Haifa University, Israel) and an international team including Jill Fielding-Wells on the impact of the COVID-19 pandemic on pedagogy in mathematics. And then there is the current ARC submission that focuses on critical aspects of mathematical thinking, including the role of social justice in mathematics-based decision-making. There remain opportunities to research and learn!

My current ZPD is now one of a mature researcher. I am now a Director of a research program within ILSTE, with a focus on STEM Education, and must accommodate all of the demands required of leadership. I have a team to mentor and lead, as well as PhD students. These create demands on my time that constitute constraints within my ZFM, as well as institutional demands that require publications be submitted to only the best journals. Increasing involvement in national and international collaborations are now an important element of my ZPA. These collaborations include both formal and informal mentoring from highly esteemed colleagues (I have been published in ESM, at last, with their support). I hope that these collaborations are also having a positive impact on the ZPA of others. My ZFM/ZPA complex is now fully directed towards research, with aspirations to excellence. This complex also overlaps with those of others in my roles of leader and mentor. I hope I am viewed more as an affordance than a constraint!

I am now fully involved in two research communities of practice: one, a national and international related to mathematics education, and the other, related to my Institution. Many of the goals are the same, but there are important differences. Each has both affordances and constraints to how I participate. I think I have now moved a little beyond novice, but it is up to others to decide if I have achieved any sort of mastery. I hope I am now achieving some aspect of my goals related to generation of new knowledge and research excellence. At the same time, facilitating the fuller participation of others into the mathematics education

community is a goal of increasing focus. That is, providing guidance than canalises the ZFM/ZPA of others – another change in identity.

Conclusion

In their research, Goos and Bennison (2019) have traced the identity trajectory of teachers in mathematics education in a manner consistent with Wenger's (1998) notion of identity-as-becoming. In this paper, I have attempted to connect this thinking to that of researcher development in mathematics education. Through this narrative, I have described a transformation of identity over time as an outcome of my participation in a range of communities – student, public servant, teacher, member of teacher professional associations, and researcher. Each participation has fostered multiple identities consistent with the practices of each community (Wenger, 1998). Entering each new community required realignment and an ongoing evaluation of whether my goals remained consistent with those of the community. Eventually I have come to participate more in some communities and less and others. Crow et al. (2017) have argued that "Key to successfully negotiating our stable selves is the reconciliation of the multiple identities which are constructed in these multiple communities of practice" (p. 268). This rings true for me as I believe I have retained the essence of each of the identities I have assumed through my career in some form, although each has come to the fore at different times – a different emphasis for person-in-practice-in-person.

So what messages do I have for researchers in mathematics education having experienced these different identities? I believe there are six, which I hope are evident in the preceding narrative:

1. *Contribute to your research community* – they will challenge you to do your best work and support you when times are tough.
2. *Work with the best in the field* – they will stretch you, bringing you forward into fuller participation in the community of mathematics educators. They will also let you know when you have more work to do before the next big step. On this point I have been lucky.
3. *Lead* - don't stand back waiting to be asked, initiate conversations about potential research ventures. Do not be afraid to bring others with you.
4. *Be wary of low hanging fruit* – test yourself, aim high in terms of international publications, keep applying for funding despite the risk of rejection. Focus on quality rather than quantity.
5. *Collaborate* – be generous with your time, there will be a point when you need to depend on others.
6. *Think nationally and internationally and not just about local demands* – there are many opportunities out there.

Through this lecture, I hope I have stimulated some thinking about the notion of a "reflective" researcher. I have two additional questions:

1. We readily place the expectation of being "reflective" on teachers. Do we do the same when considering the development of researcher identity?
2. What will be my/your next transformation of identity?

I finish with another quote drawn from culture, this time Andy Warhol.

> When people are ready to, they change. They never do it before then, and sometimes they die before they get around to it. You can't make them change if they don't want to, just like when they do want to, you can't stop them.

How much you wish to change very much depends on you.

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